MARK D. FAIRCHILD & YUTA ASANO

CUSTOM COLOR MATCHING FUNCTIONS:
EXTENDING THE CIE 2006 MODEL

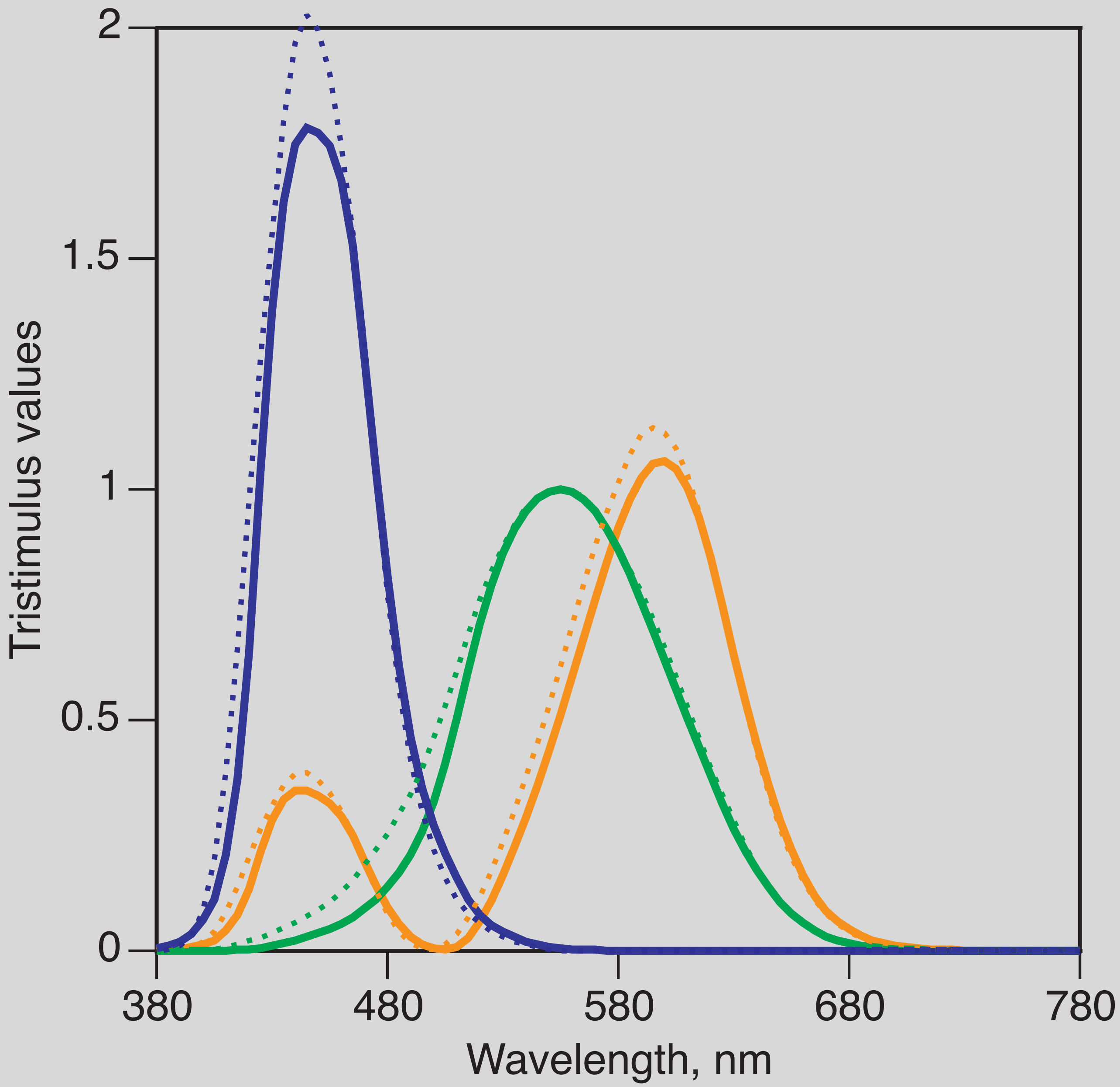
TWO BIG QUESTIONS…

WHAT DO YOU SEE?

DO YOU SEE WHAT I SEE?

CIE COLORIMETRY

- 1931 & 1964, 2- & 10-deg.



INDIVIDUALS

- Stiles & Burch (1959)

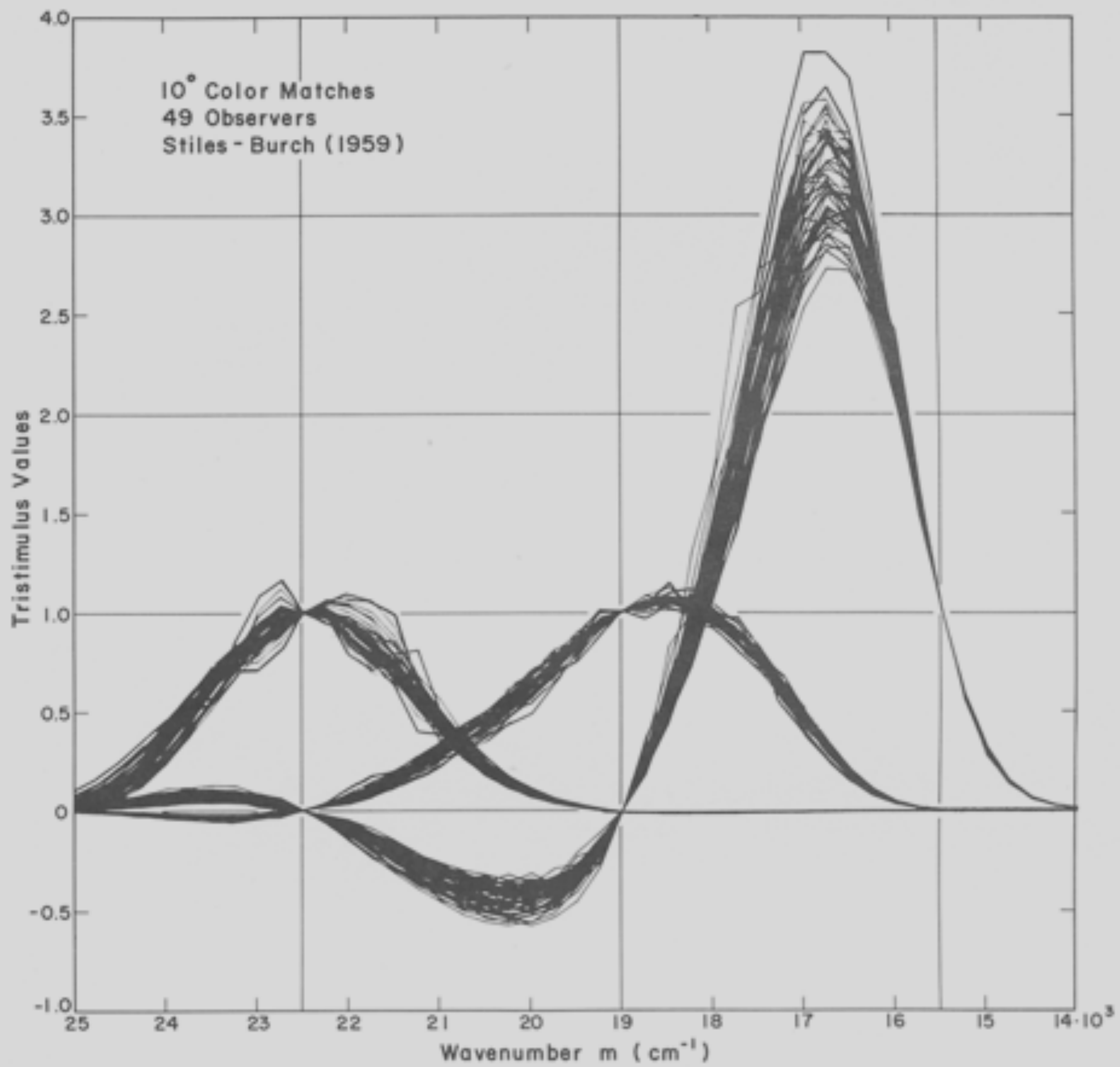


Fig. 3(5.5.6). 10° color-matching functions of 49 observers participating in the Stiles–Burch (1959) experiment. All functions refer to primary stimuli at $m_R = 15{,}500$, $m_G = 19{,}000$, and $m_B = 22{,}500$ cm^{-1}.

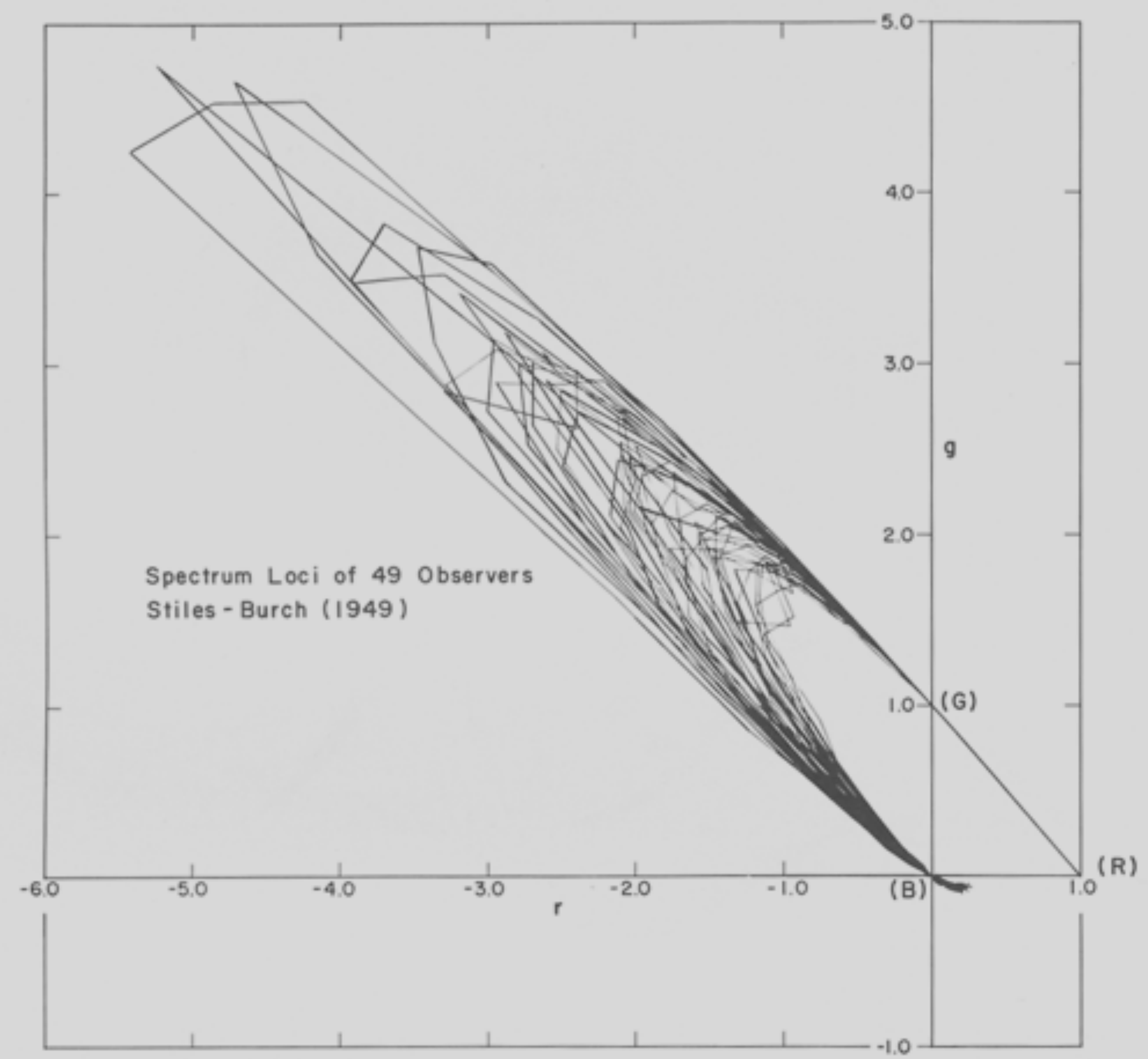


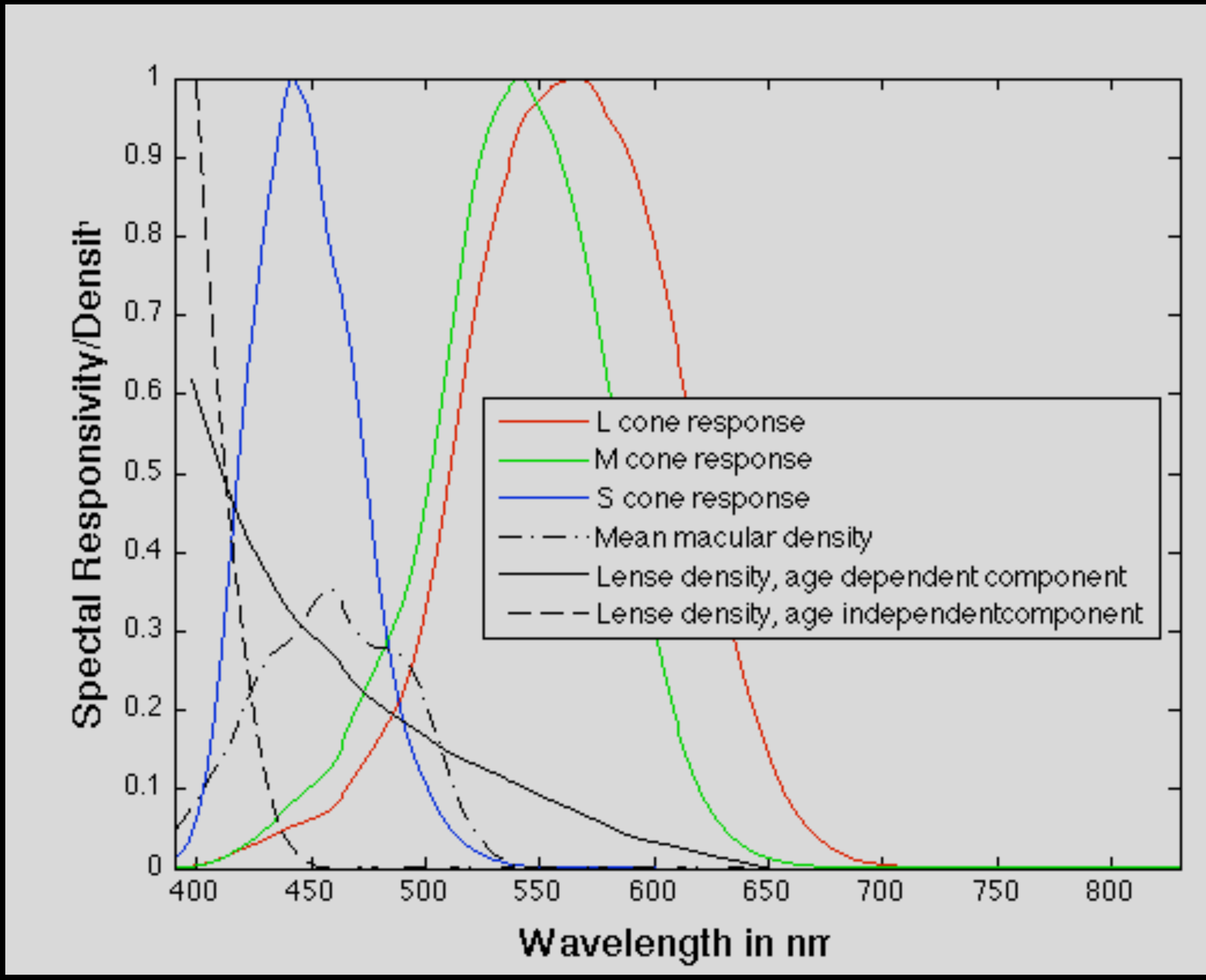
Fig. 4(5.5.6). (r, g)-chromaticity diagram of Stiles–Burch (1959) 10° color-matching investigation showing the spectrum loci of 49 observers.

MONTE CARLO …

- Fairchild & Heckaman (2013) and *in press*
- Build individual observers
- Statistically analyze and create “Nimeroff” system

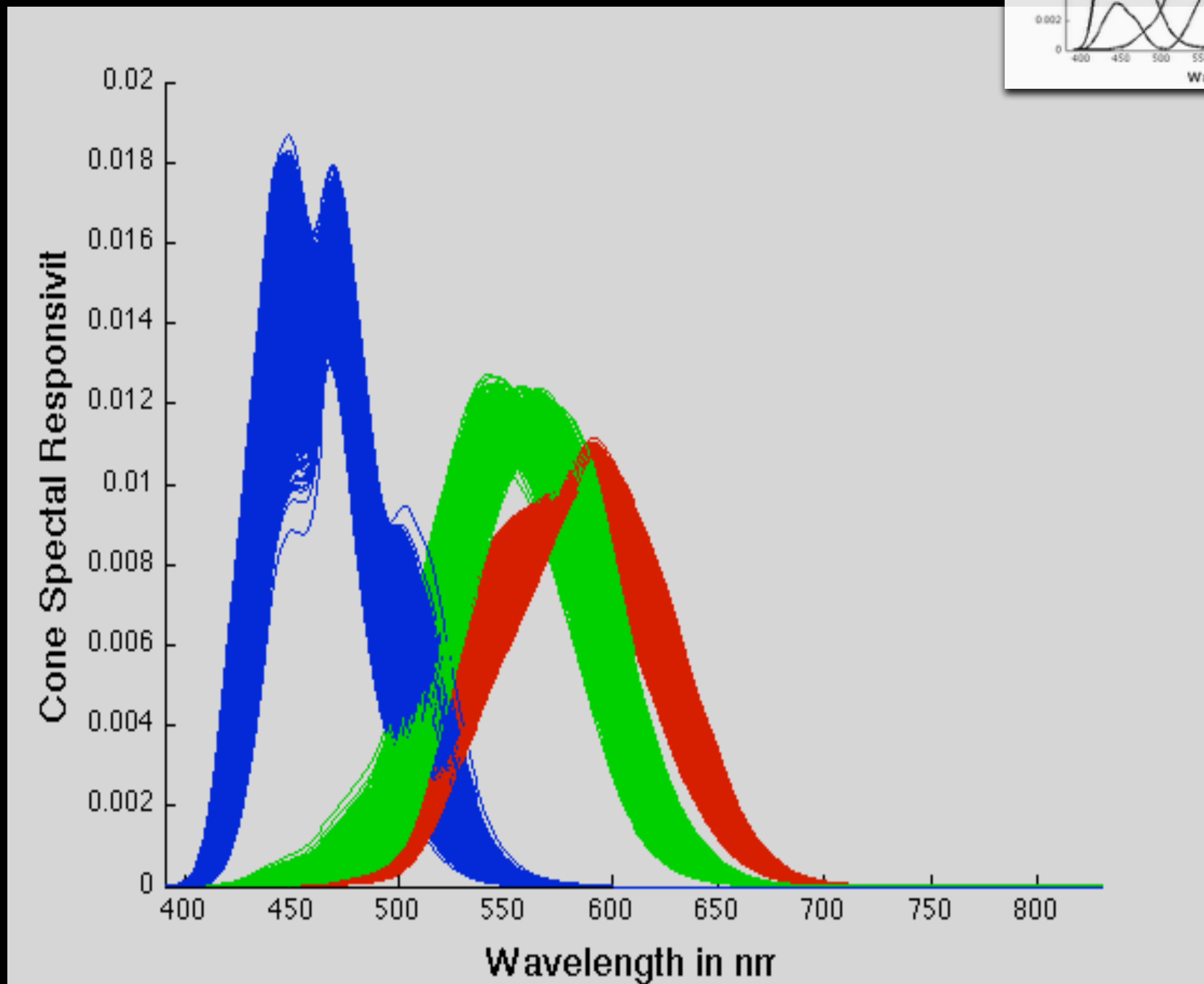
RANDOMLY SELECT

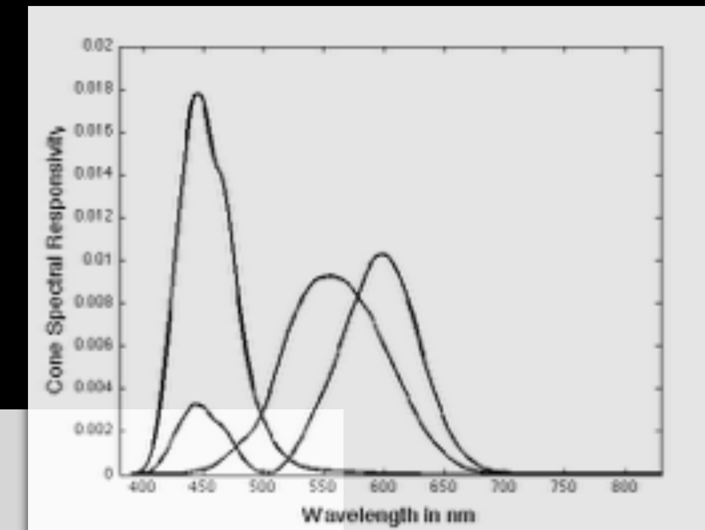
- Lens (Density)
- Macula (Density)
- L, M, & S Cones (Shift)



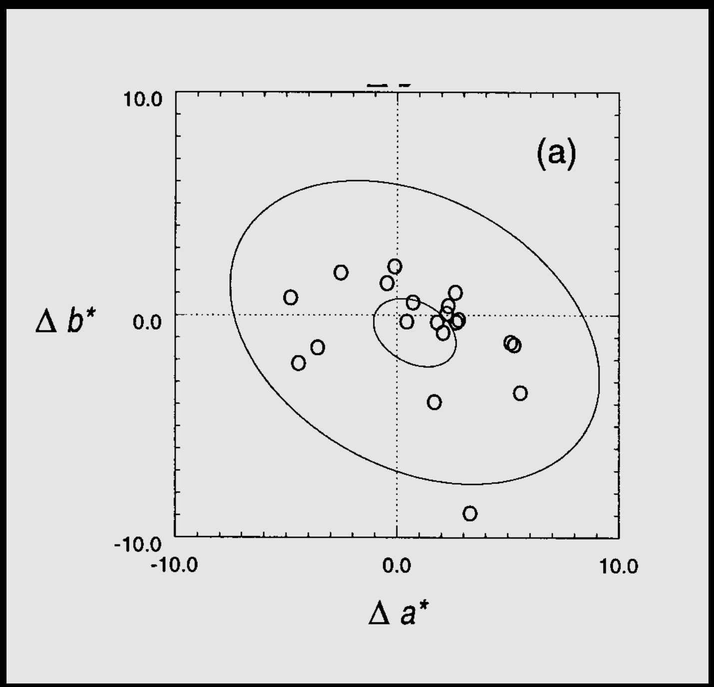
- Build Cone Fundamentals
- Compute Other CMFs

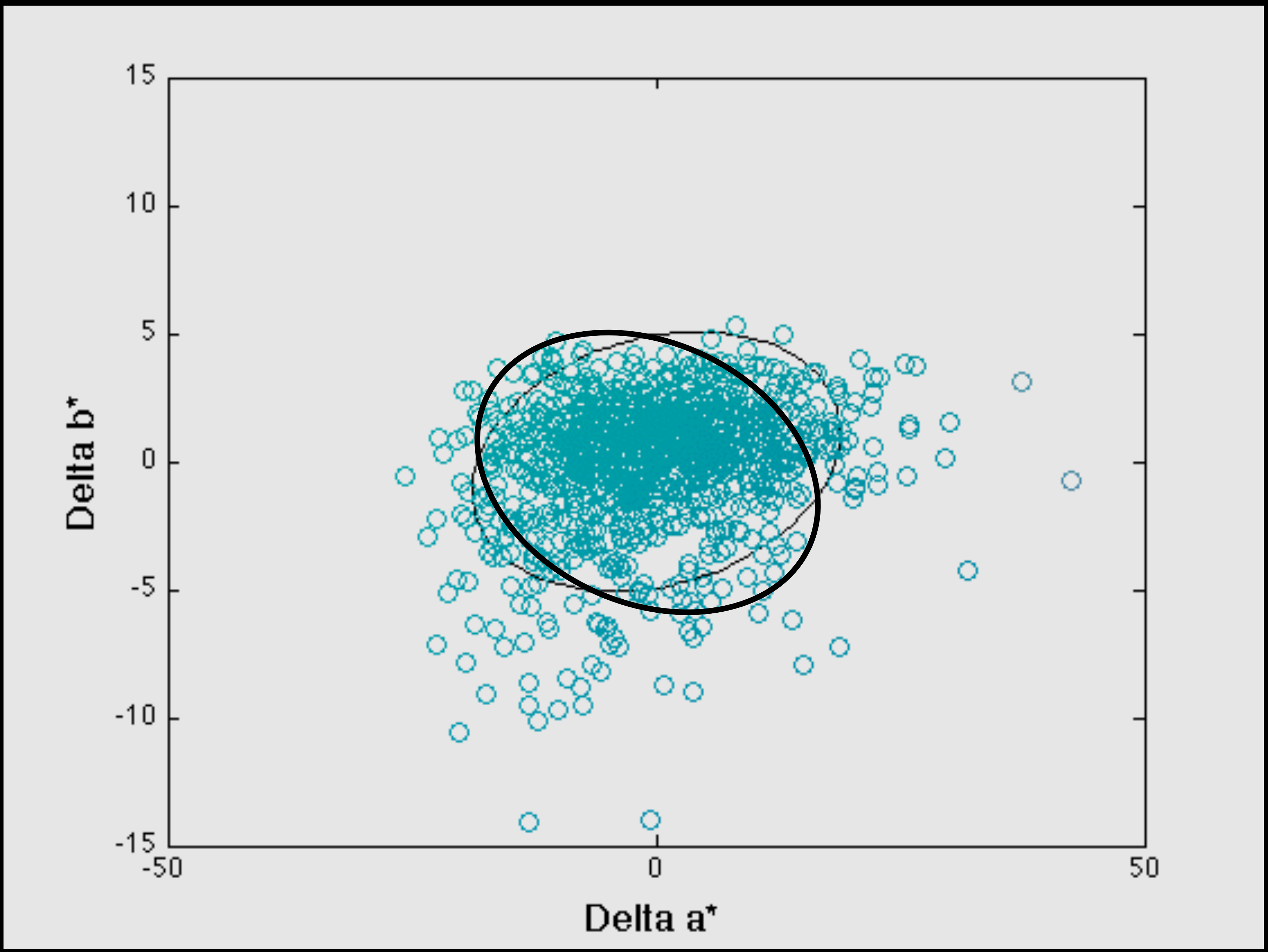
1000 OBSERVERS





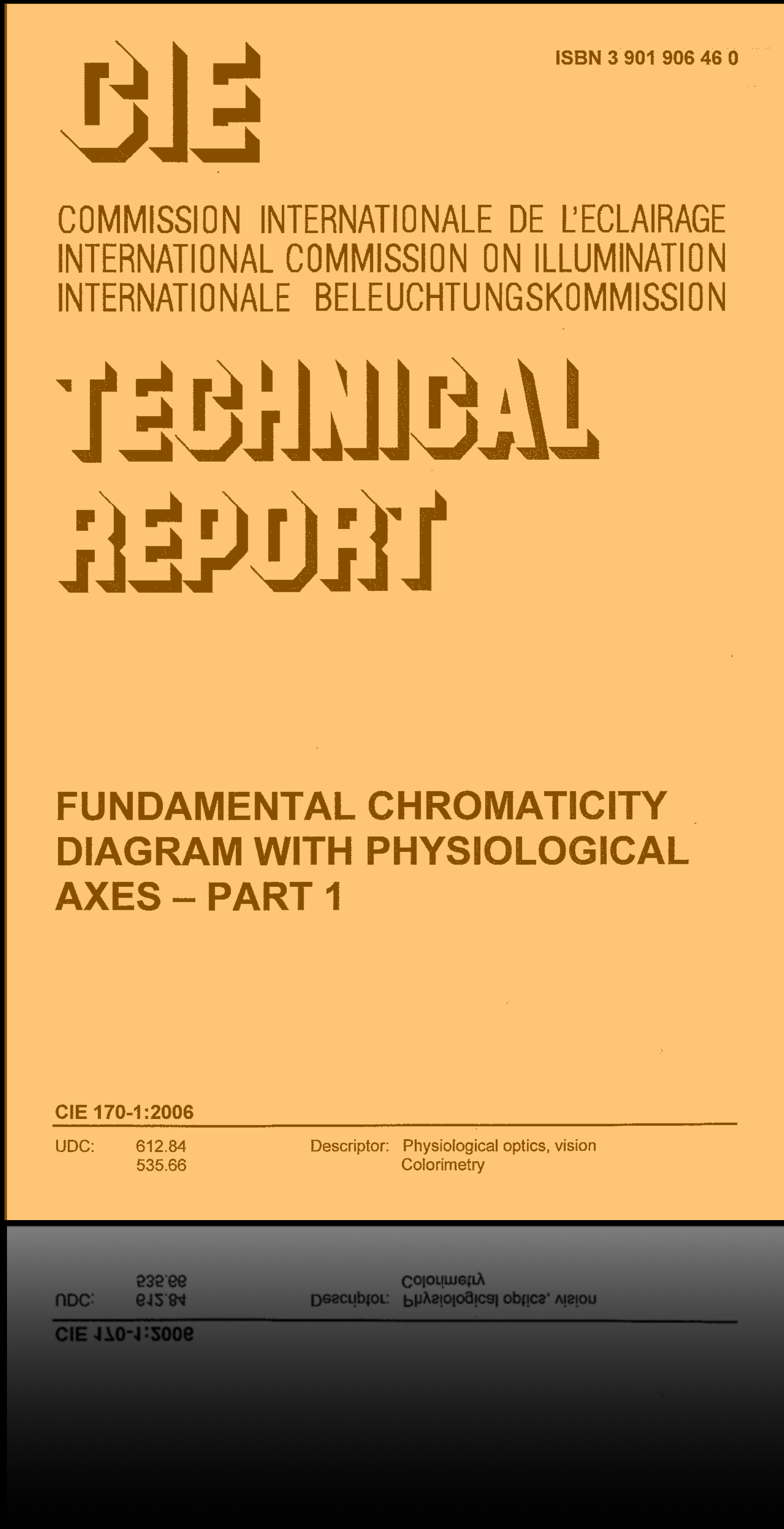
ALFVIN & FAIRCHILD CYAN SAMPLE





CIE 2006 APPROACH

- TC1-36
- Fundamental Chromaticity Diagram with Physiological Axes
- CIE 170-1 (2006)
- Compute cone responsivities (LMS) as a function of age and field size



CIE 2006

$$\bar{l}(\lambda) = \alpha_{i,l}(\lambda) \cdot 10^{\left[-D_{\tau,\max,macula} \cdot D_{macula,relative}(\lambda) - D_{\tau,ocul}(\lambda)\right]}$$

$$\overline{m}(\lambda) = \alpha_{i,m}(\lambda) \cdot 10^{\left[-D_{\tau,\max,macula} \cdot D_{macula,relative}(\lambda) - D_{\tau,ocul}(\lambda)\right]}$$

$$\bar{s}(\lambda) = \alpha_{i,s}(\lambda) \cdot 10^{\left[-D_{\tau,\max,macula} \cdot D_{macula,relative}(\lambda) - D_{\tau,ocul}(\lambda)\right]}$$

CIE 2006

Cone Absorptivity Spectra

f(field size)

$$\bar{l}(\lambda) = \alpha_{i,l}(\lambda) \cdot 10^{\left[-D_{\tau,\max,macula} \cdot D_{macula,relative}(\lambda) - D_{\tau,ocul}(\lambda)\right]}$$

$$\bar{m}(\lambda) = \alpha_{i,m}(\lambda) \cdot 10^{\left[-D_{\tau,\max,macula} \cdot D_{macula,relative}(\lambda) - D_{\tau,ocul}(\lambda)\right]}$$

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CIE 2006

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$$\bar{s}(\lambda) = \alpha_{i,s}(\lambda) \cdot 10^{\left[-D_{\tau,\max,macula} \cdot D_{macula,relative}(\lambda) - D_{\tau,ocul}(\lambda)\right]}$$

Macular Density

f(field size)

CIE 2006

$$\bar{l}(\lambda) = \alpha_{i,l}(\lambda) \cdot 10^{\left[-D_{\tau,\max,macula} \cdot D_{macula,relative}(\lambda) - D_{\tau,ocul}(\lambda)\right]}$$

$$\bar{m}(\lambda) = \alpha_{i,m}(\lambda) \cdot 10^{\left[-D_{\tau,\max,macula} \cdot D_{macula,relative}(\lambda) - D_{\tau,ocul}(\lambda)\right]}$$

$$\bar{s}(\lambda) = \alpha_{i,s}(\lambda) \cdot 10^{\left[-D_{\tau,\max,macula} \cdot D_{macula,relative}(\lambda) - D_{\tau,ocul}(\lambda)\right]}$$

Cone Absorptivity Spectra

f(field size)

Ocular Media Density

f(age)

Macular Density

f(field size)

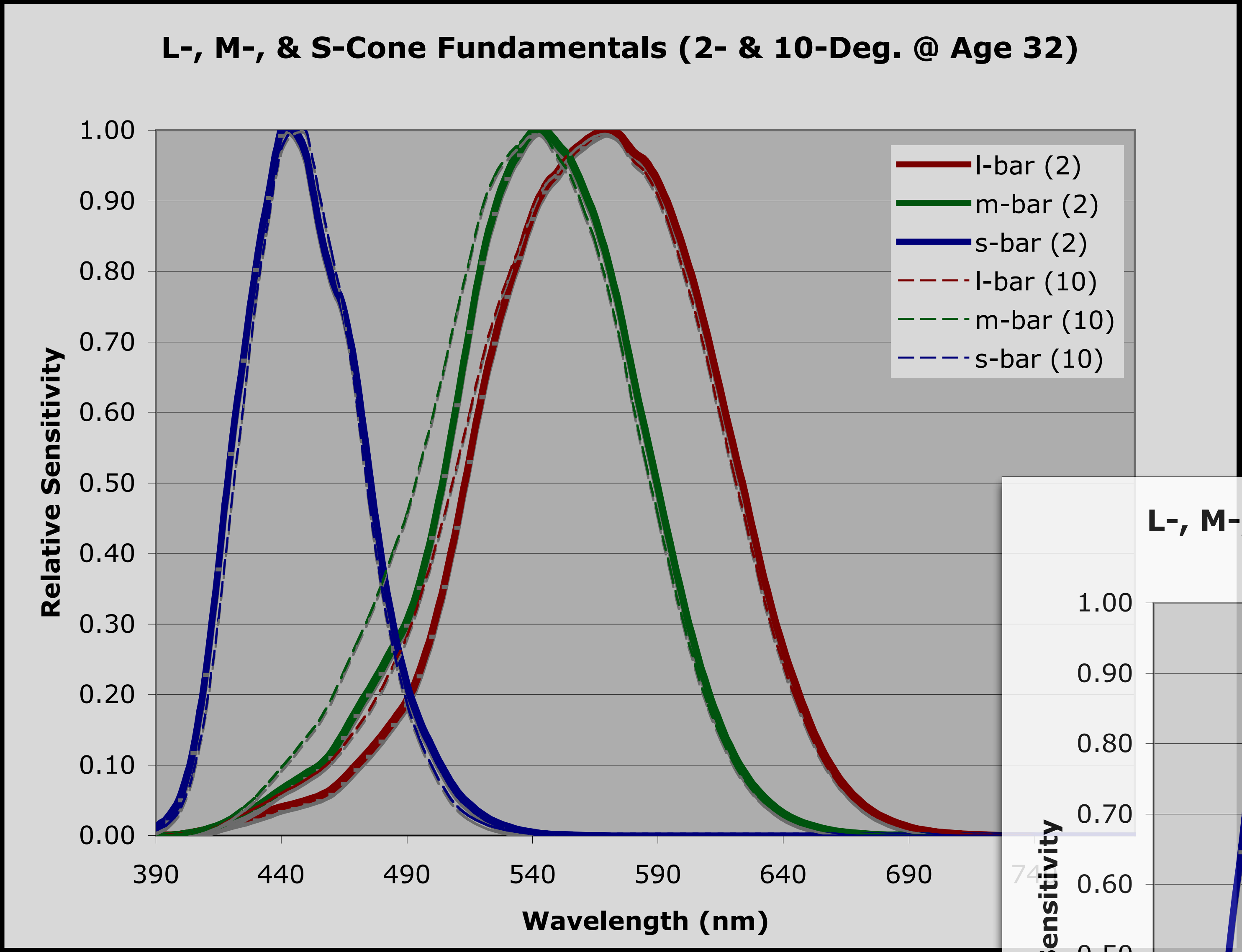
MEAN OBSERVERS

MEAN OBSERVERS

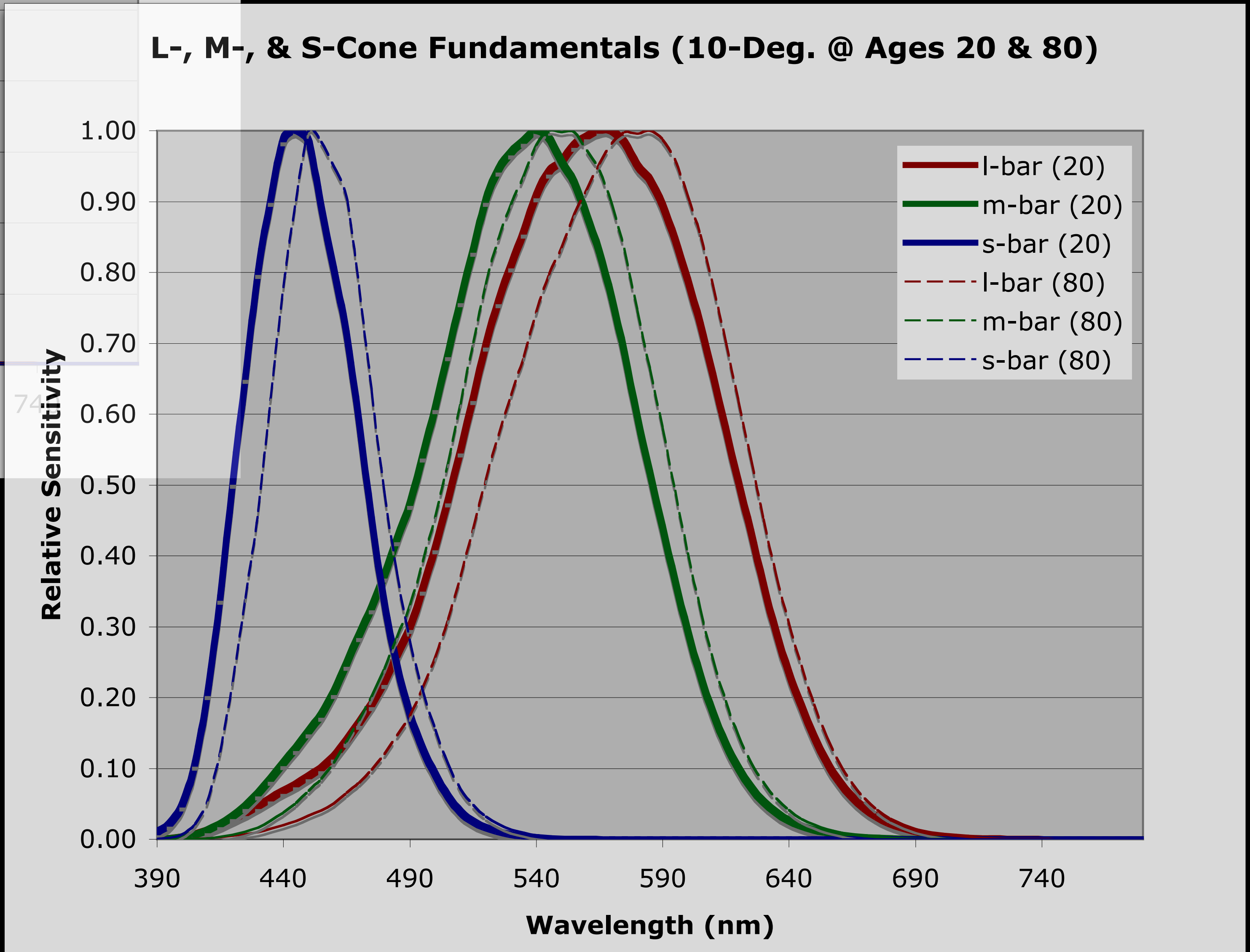
MEAN OBSERVERS

MEAN OBSERVERS

EXAMPLES:

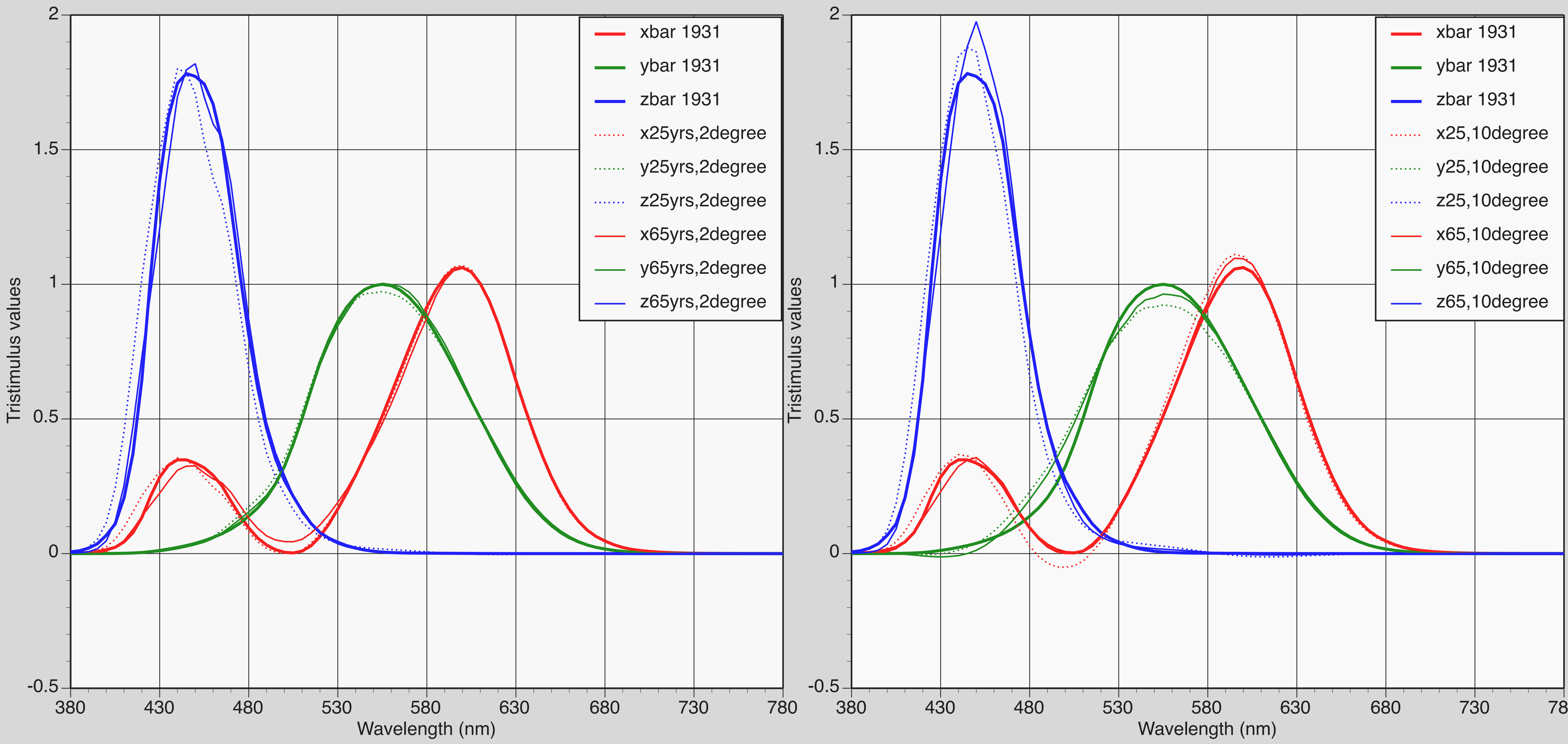


L-, M-, & S-Cone Fundamentals (2- & 10-Deg. @ Age 32)
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
Relative Sensitivity
390
440
490
540
590
640
690
Wavelength (nm)
l-bar (2)
m-bar (2)
s-bar (2)
l-bar (10)
m-bar (10)
s-bar (10)



L-, M-, & S-Cone Fundamentals (10-Deg. @ Ages 20 & 80)
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
Relative Sensitivity
390
440
490
540
590
640
690
740
Wavelength (nm)
l-bar (20)
m-bar (20)
s-bar (20)
l-bar (80)
m-bar (80)
s-bar (80)

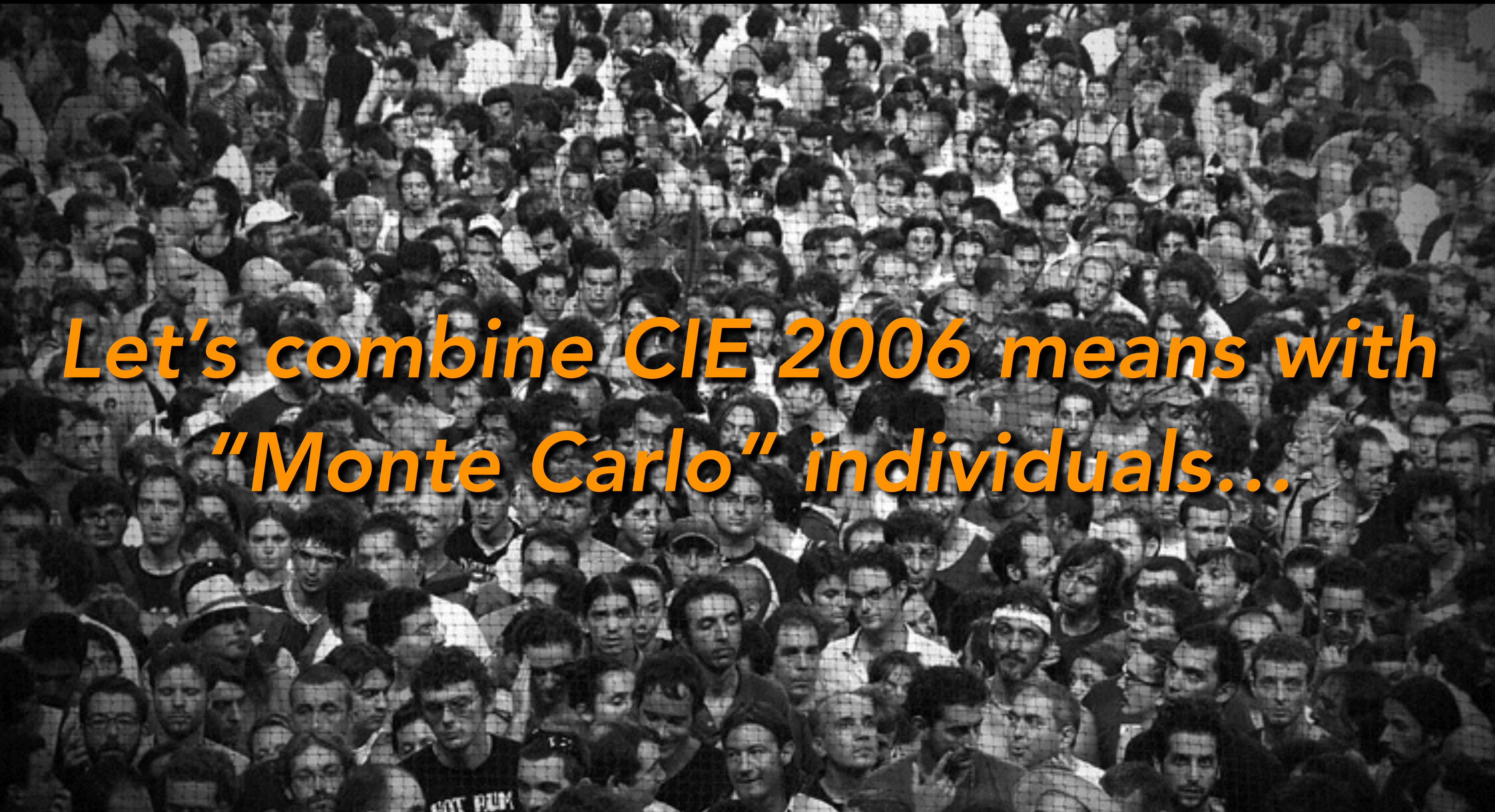
EXAMPLES:



xbar 1931
ybar 1931
zbar 1931
x25yrs,2degree
y25yrs,2degree
z25yrs,2degree
x65yrs,2degree
y65yrs,2degree
z65yrs,2degree
Tristimulus values
Wavelength (nm)
2
1.5
1
0.5
0
-0.5
380
430
480
530
580
630
680
730
780
xbar 1931
ybar 1931
zbar 1931
x25,10degree
y25,10degree
z25,10degree
x65,10degree
y65,10degree
z65,10degree
Tristimulus values
Wavelength (nm)

NOT INDIVIDUALS

NOT INDIVIDUALS



Let’s combine CIE 2006 means with “Monte Carlo” individuals…

ASANO MODEL

- Start with CIE 2006 mean observers
- Perturb with individual variations in physiological components
- Create individual color matching functions
- Monte Carlo or measurement driven

CIE 2006 + INDIVIDUALS

$$lms\text{-}CMFs = f(age, fs, d_{lens}, d_{macula}, d_L, d_M, d_S, s_L, s_M, s_S)$$

Input: age, field size, 8 physiological parameters

Output: lms-CMFs (= Cone Fundamentals)

Standard deviations derived from past studies, then scaled to fit a set of color matching data

OBTAINED STANDARD DEVIATIONS

LENS [%]	MACULA [%]	DENSITY [%]			λ_{MAX} SHIFT [nm]		
		L	M	S	L	M	S
18.7	36.5	9.0	9.0	7.4	2.0	1.5	1.3

CIE 2006 + INDIVIDUALS

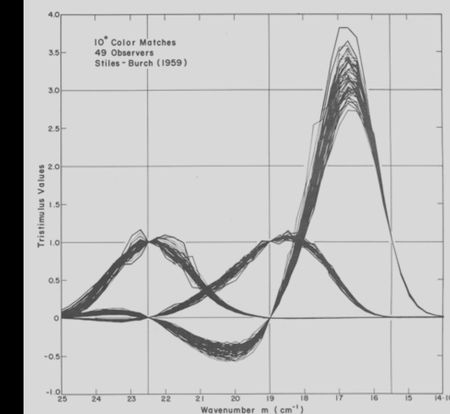


Fig. 3(5.5.6). 10° color-matching functions of 49 observers participating in the Stiles–Burch (1959) experiment. All functions refer to primary stimuli at $m_R = 15{,}500$, $m_G = 19{,}000$, and $m_B = 22{,}500\ cm^{-1}$.

- Stiles & Burch 49 Observers

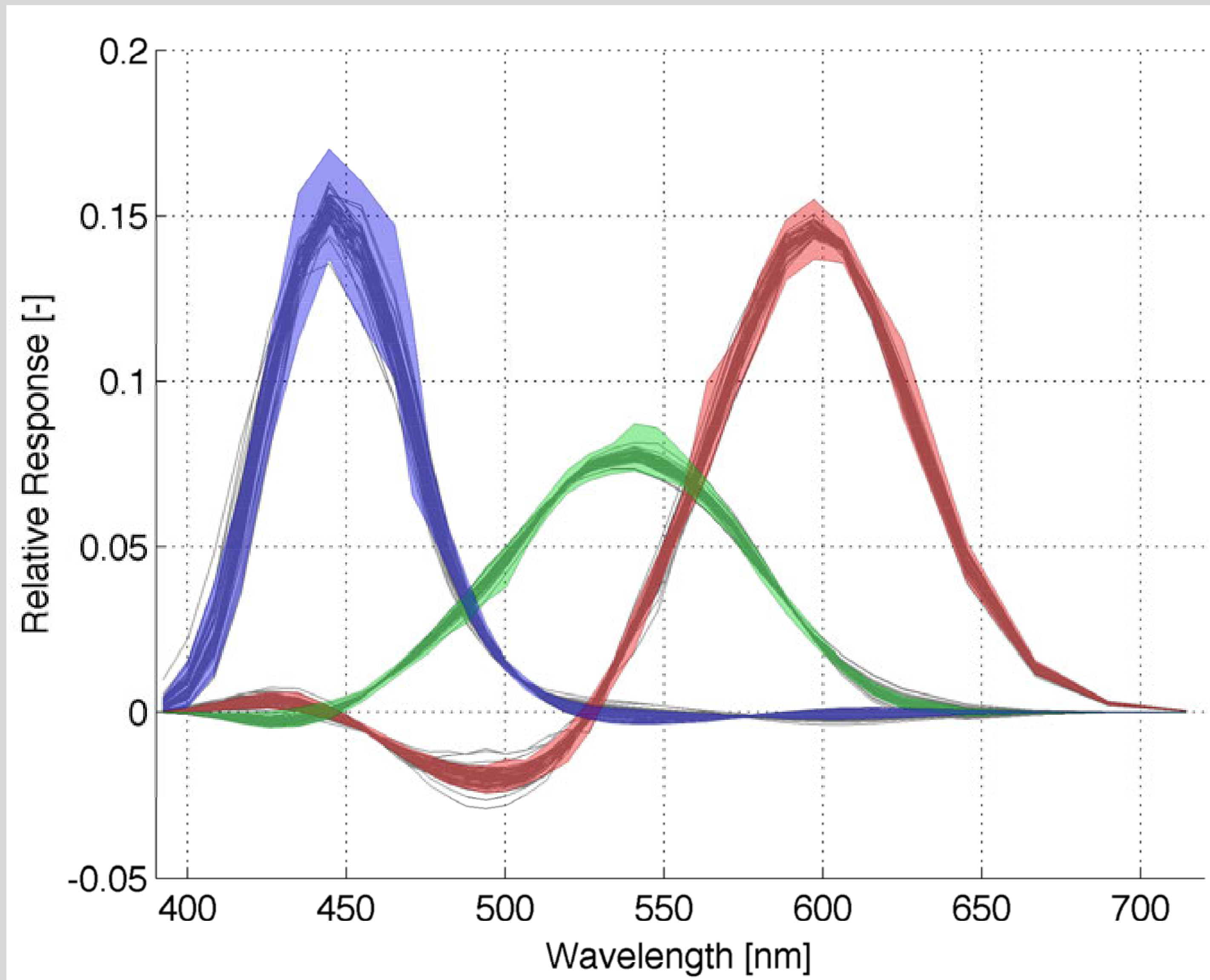
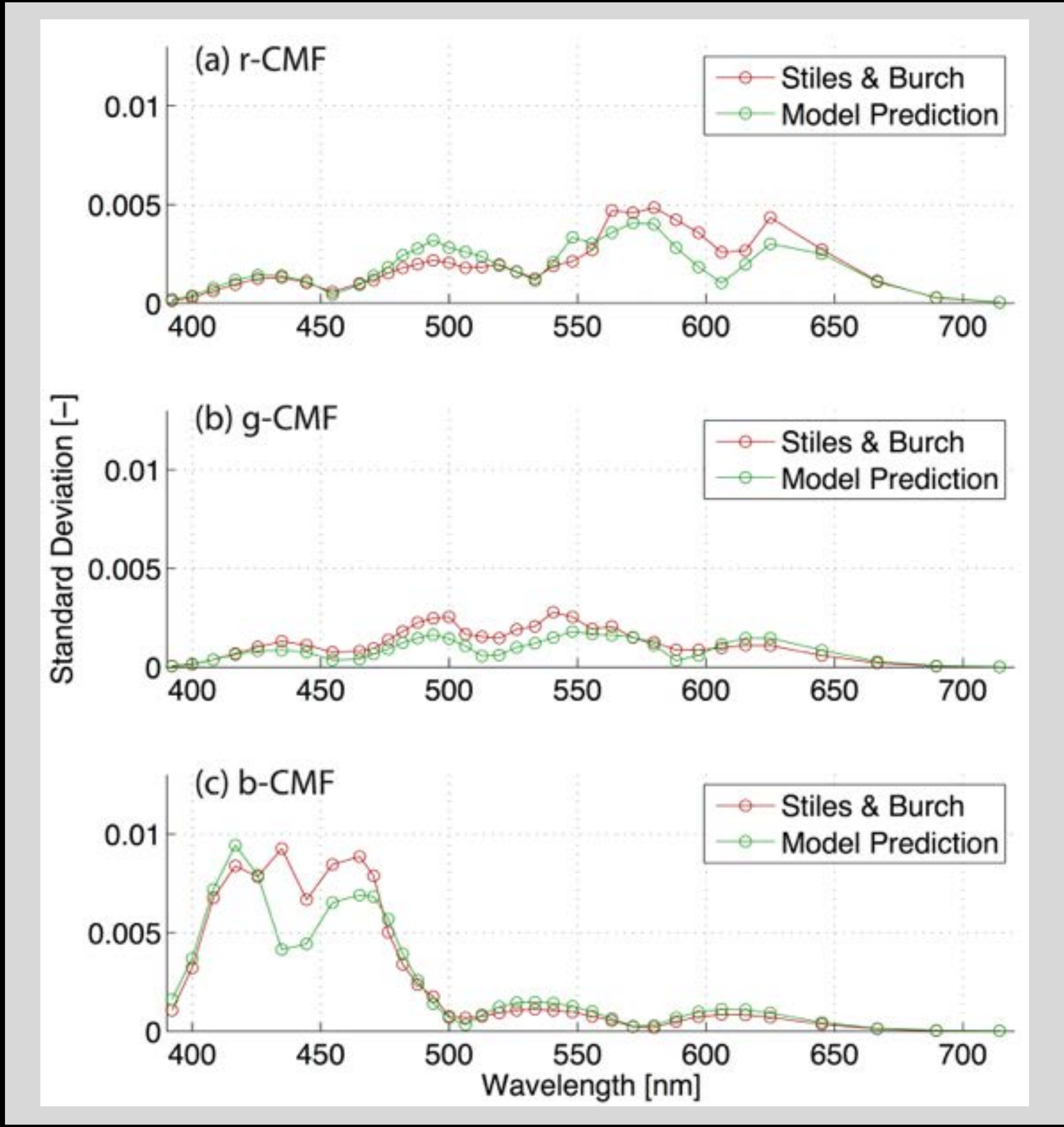


Fig. 3.12 – 49 sets of rgb-CMFs generated by the proposed observer model (gray lines) aiming to predict the Stiles and Burch's experiment results. The maxima and minima of 49 sets of CMFs for the Stiles and Burch's experiment participants are superimposed as color-shaded areas. All the CMFs are normalized to equal area.

CIE 2006 + INDIVIDUALS



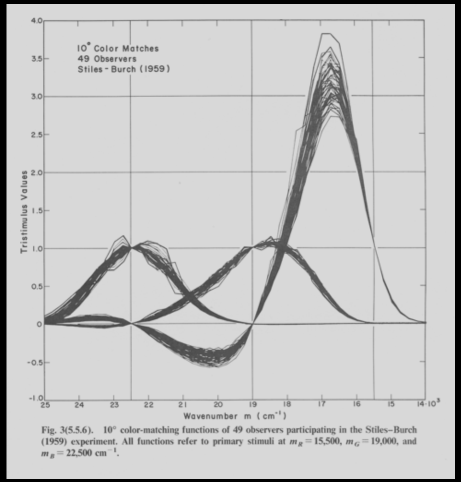


Fig. 3(5.5.6). 10° color-matching functions of 49 observers participating in the Stiles–Burch (1959) experiment. All functions refer to primary stimuli at $m_R = 15{,}500$, $m_G = 19{,}000$, and $m_B = 22{,}500$ cm^{-1}.

NO LONGER MEAN

Nice, Individual, Observers

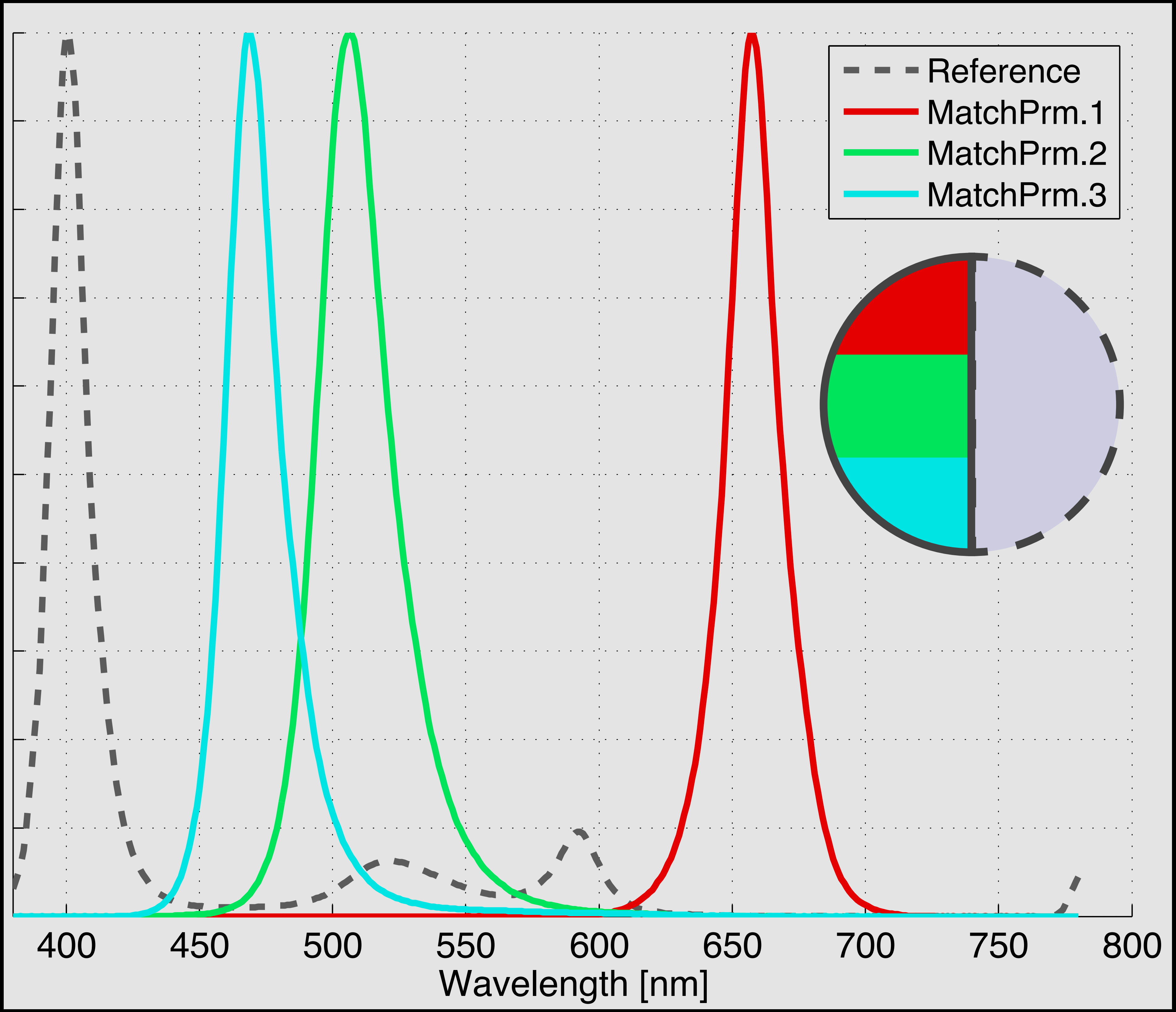
INDIVIDUALIZED COLORIMETRY

- Observer Calibrator (5 Matches)
- Individual Parameters
- Asano Model
- Individual (Customized) Color Matching Functions

OBSERVER METAMERISM DEMO

technicolor

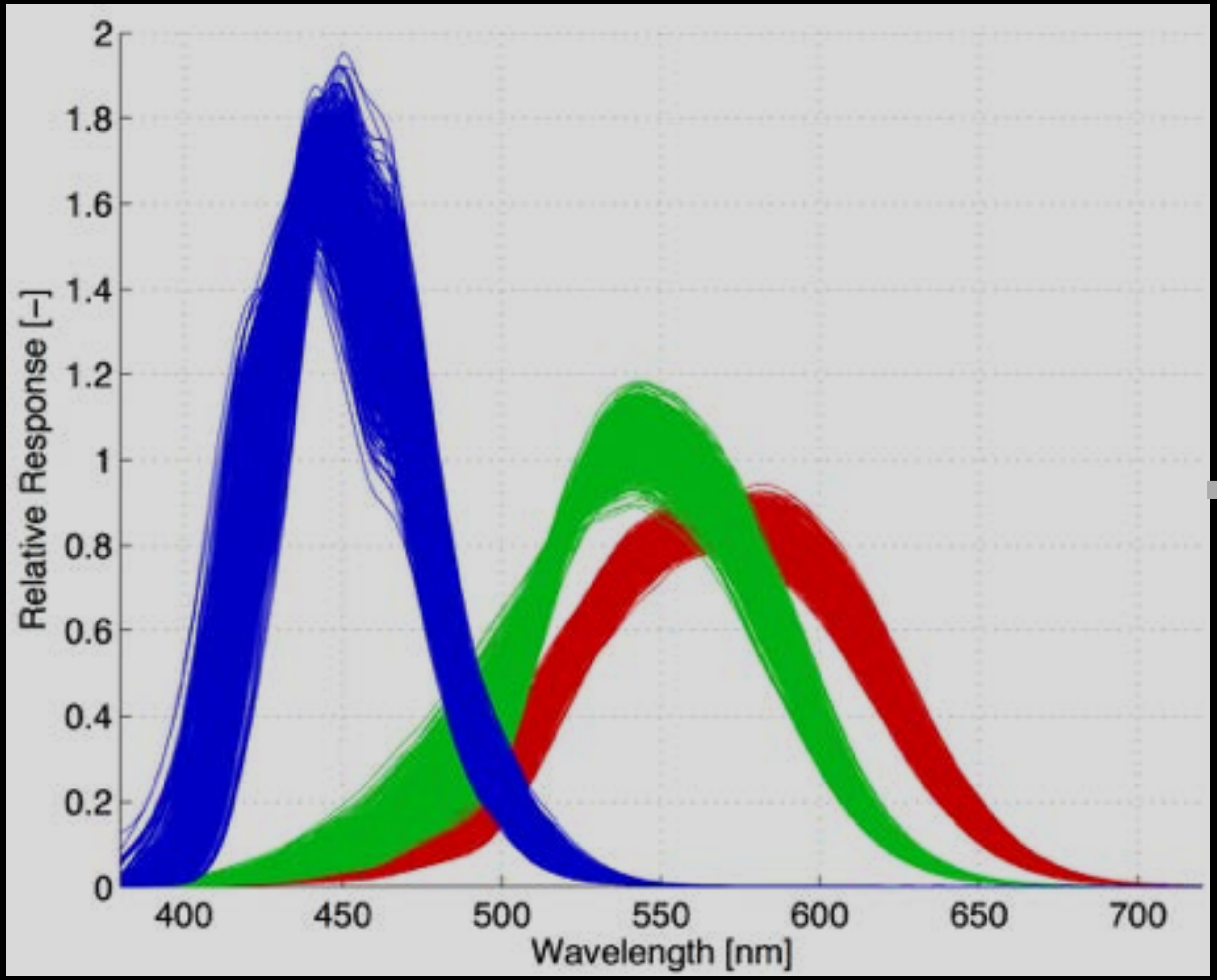
Spectra Generated from Different LEDs to Magnify Inter-Observer Variability

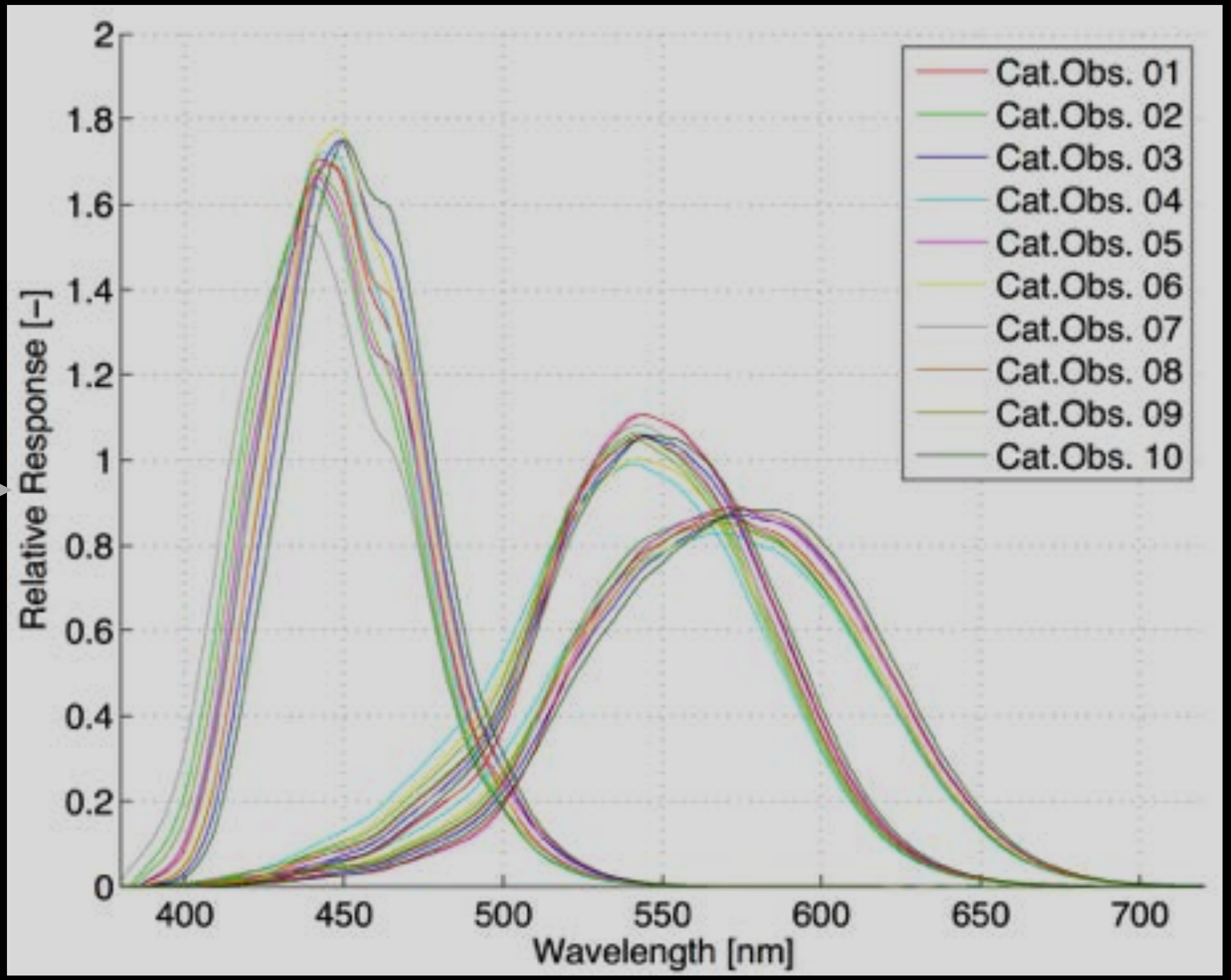


CATEGORICAL OBSERVERS

STEP 1: GENERATE 10,000 CMFS BY INDIVIDUAL OBSERVER MODEL + MONTE CARLO SIMULATION

STEP 2: CLUSTER ANALYSIS





CATEGORIES

- Following on the work of Sarkar et al.

Tab. 3.8 – Ages and eight physiological parameters for the first ten categorical observers.

Cat. Obs. ID	1	2	3	4	5	6	7	8	9	10
Age	38	30	56	33	38	45	31	51	35	68
Lens Density [%]	0	-22.9	17.0	-8.3	1.6	7.0	-34.0	15.0	-18.3	10.9
Macula Density [%]	0	7.0	-11.0	-43.6	54.7	-35.3	36.3	30.8	-11.9	-16.0
Density in L [%]	0	-11.1	0.6	5.9	3.7	4.8	7.3	2.4	-2.4	0.7
Density in M [%]	0	-5.0	-5.5	4.5	16.1	11.6	7.4	-8.7	-7.0	-10.3
Density in S [%]	0	7.6	-1.0	0.2	-1.8	-4.5	-4.6	0.0	-9.9	9.3
Shift in L [nm]	0	-0.1	0.9	-1.0	1.1	-0.6	-0.6	0.5	0.3	0.7
Shift in M [nm]	0	0.3	0.5	-1.4	-1.1	-1.3	0.1	0.1	-0.6	0.4
Shift in S [nm]	0	-0.8	0.7	0.1	0.2	-0.1	0.8	0.1	-0.1	0.4

CATEGORIES

- Following on the work of Sarkar *et al.*

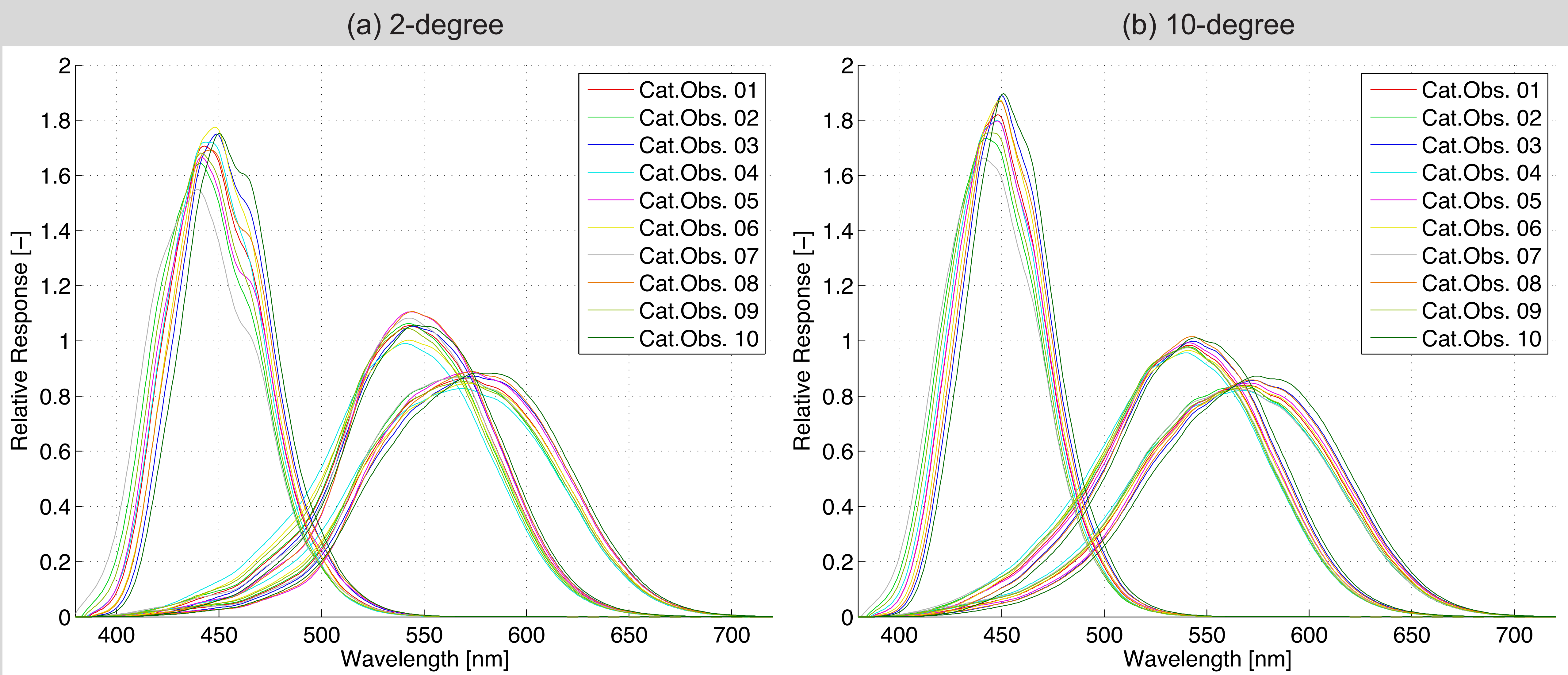
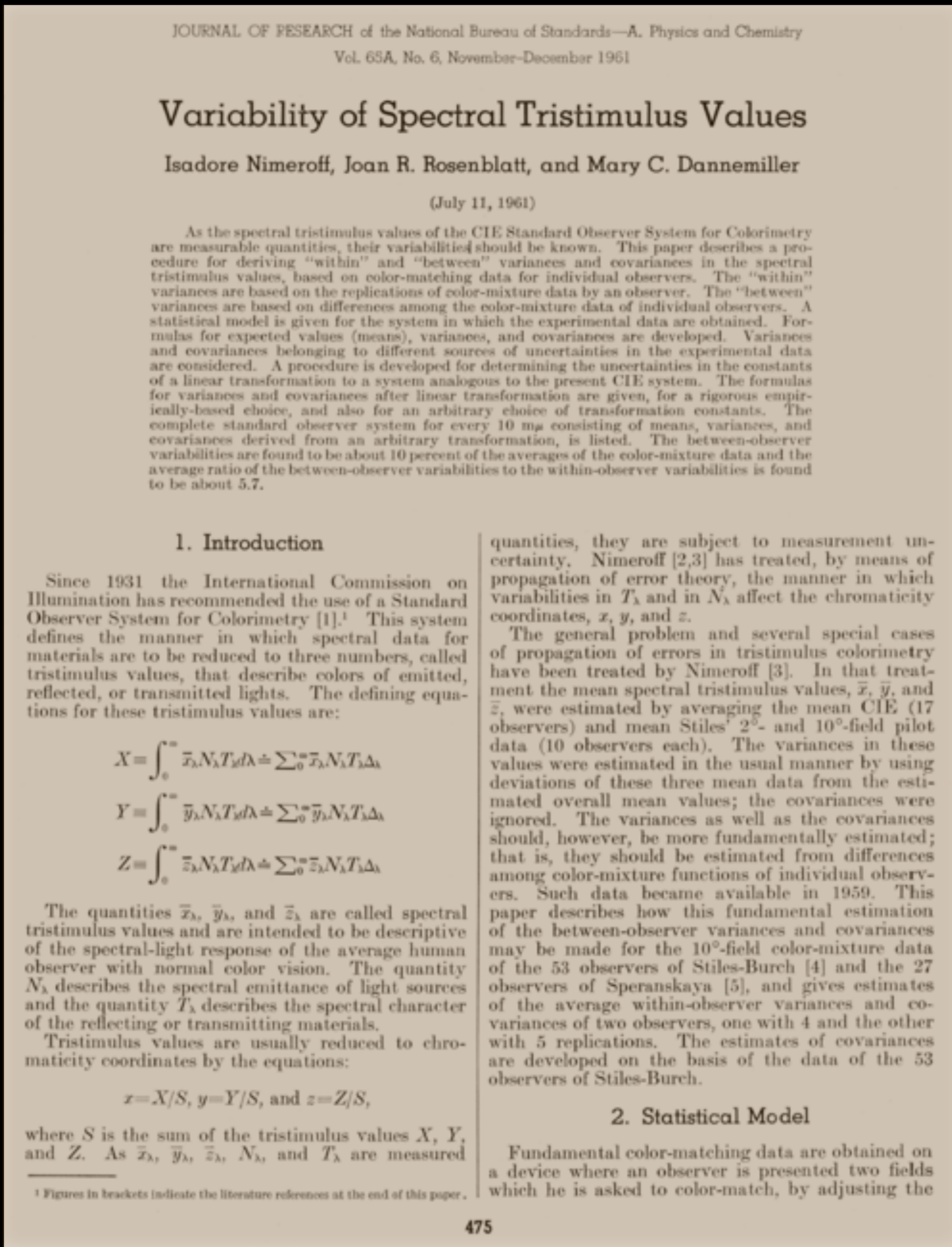


Fig. 3.16 – lms-CMFs (cone fundamentals) of the first ten categorical observers for a field size of 2° (a) and 10° (b). Each function is area-normalized.

CONFIDENCE ELLIPSOIDS


JOURNAL OF RESEARCH of the National Bureau of Standards—A. Physics and Chemistry
Vol. 65A, No. 6, November–December 1961

Variability of Spectral Tristimulus Values

Isadore Nimeroff, Joan R. Rosenblatt, and Mary C. Dannemiller

(July 11, 1961)

As the spectral tristimulus values of the CIE Standard Observer System for Colorimetry are measurable quantities, their variabilities should be known. This paper describes a procedure for deriving "within" and "between" variances and covariances in the spectral tristimulus values, based on color-matching data for individual observers. The "within" variances are based on the replications of color-mixture data by an observer. The "between" variances are based on differences among the color-mixture data of individual observers. A statistical model is given for the system in which the experimental data are obtained. Formulas for expected values (means), variances, and covariances are developed. Variances and covariances belonging to different sources of uncertainties in the experimental data are considered. A procedure is developed for determining the uncertainties in the constants of a linear transformation to a system analogous to the present CIE system. The formulas for variances and covariances after linear transformation are given, for a rigorous empirically-based choice, and also for an arbitrary choice of transformation constants. The complete standard observer system for every 10 mμ consisting of means, variances, and covariances derived from an arbitrary transformation, is listed. The between-observer variabilities are found to be about 10 percent of the averages of the color-mixture data and the average ratio of the between-observer variabilities to the within-observer variabilities is found to be about 5.7.

1. Introduction

Since 1931 the International Commission on Illumination has recommended the use of a Standard Observer System for Colorimetry [1].[1] This system defines the manner in which spectral data for materials are to be reduced to three numbers, called tristimulus values, that describe colors of emitted, reflected, or transmitted lights. The defining equations for these tristimulus values are:

$$X=\int_0^\infty \bar{x}_\lambda N_\lambda T_\lambda d\lambda \doteq \sum_0^\infty \bar{x}_\lambda N_\lambda T_\lambda \Delta_\lambda$$

$$Y=\int_0^\infty \bar{y}_\lambda N_\lambda T_\lambda d\lambda \doteq \sum_0^\infty \bar{y}_\lambda N_\lambda T_\lambda \Delta_\lambda$$

$$Z=\int_0^\infty \bar{z}_\lambda N_\lambda T_\lambda d\lambda \doteq \sum_0^\infty \bar{z}_\lambda N_\lambda T_\lambda \Delta_\lambda$$

The quantities $\bar{x}_\lambda$, $\bar{y}_\lambda$, and $\bar{z}_\lambda$ are called spectral tristimulus values and are intended to be descriptive of the spectral-light response of the average human observer with normal color vision. The quantity N_λ describes the spectral emittance of light sources and the quantity T_λ describes the spectral character of the reflecting or transmitting materials.

Tristimulus values are usually reduced to chromaticity coordinates by the equations:

$$x=X/S,\ y=Y/S,\ \text{and}\ z=Z/S,$$

where S is the sum of the tristimulus values X, Y, and Z. As $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$, N_λ, and T_λ are measured quantities, they are subject to measurement uncertainty. Nimeroff [2,3] has treated, by means of propagation of error theory, the manner in which variabilities in T_λ and in N_λ affect the chromaticity coordinates, x, y, and z.

The general problem and several special cases of propagation of errors in tristimulus colorimetry have been treated by Nimeroff [3]. In that treatment the mean spectral tristimulus values, $\bar{x}$, $\bar{y}$, and $\bar{z}$, were estimated by averaging the mean CIE (17 observers) and mean Stiles' 2°- and 10°-field pilot data (10 observers each). The variances in these values were estimated in the usual manner by using deviations of these three mean data from the estimated overall mean values; the covariances were ignored. The variances as well as the covariances should, however, be more fundamentally estimated; that is, they should be estimated from differences among color-mixture functions of individual observers. Such data became available in 1959. This paper describes how this fundamental estimation of the between-observer variances and covariances may be made for the 10°-field color-mixture data of the 53 observers of Stiles-Burch [4] and the 27 observers of Speranskaya [5], and gives estimates of the average within-observer variances and covariances of two observers, one with 4 and the other with 5 replications. The estimates of covariances are developed on the basis of the data of the 53 observers of Stiles-Burch.

2. Statistical Model

Fundamental color-matching data are obtained on a device where an observer is presented two fields which he is asked to color-match, by adjusting the

[1] Figures in brackets indicate the literature references at the end of this paper.

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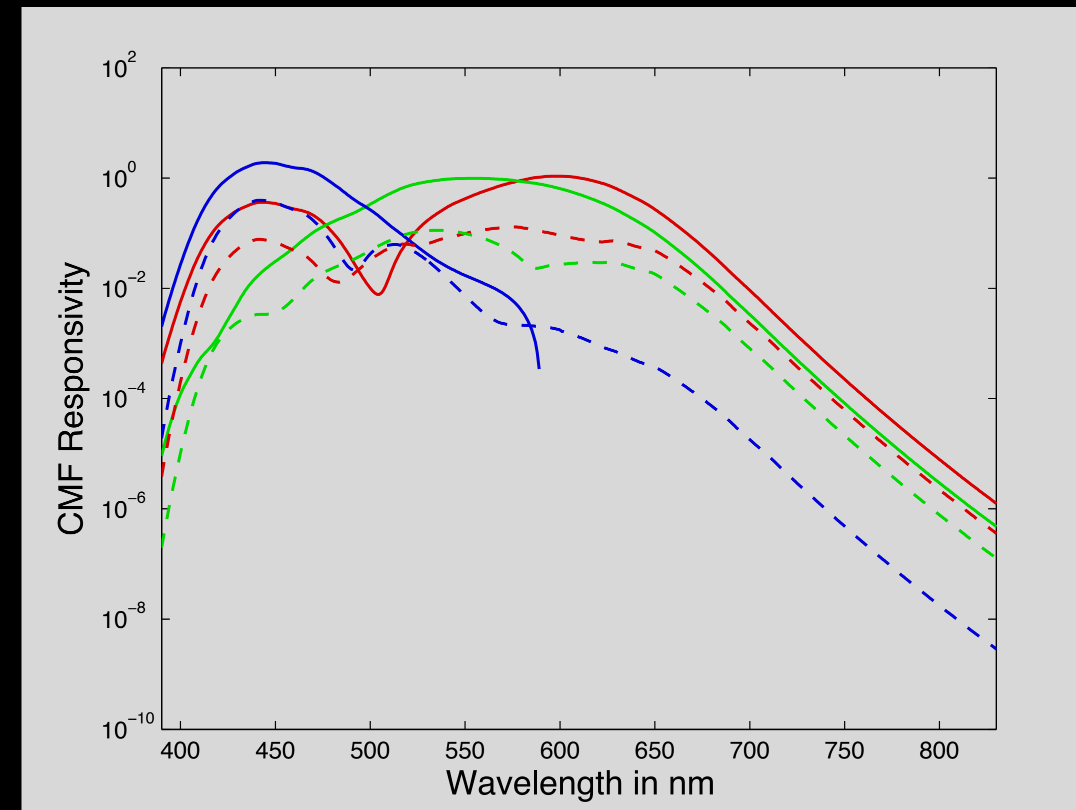
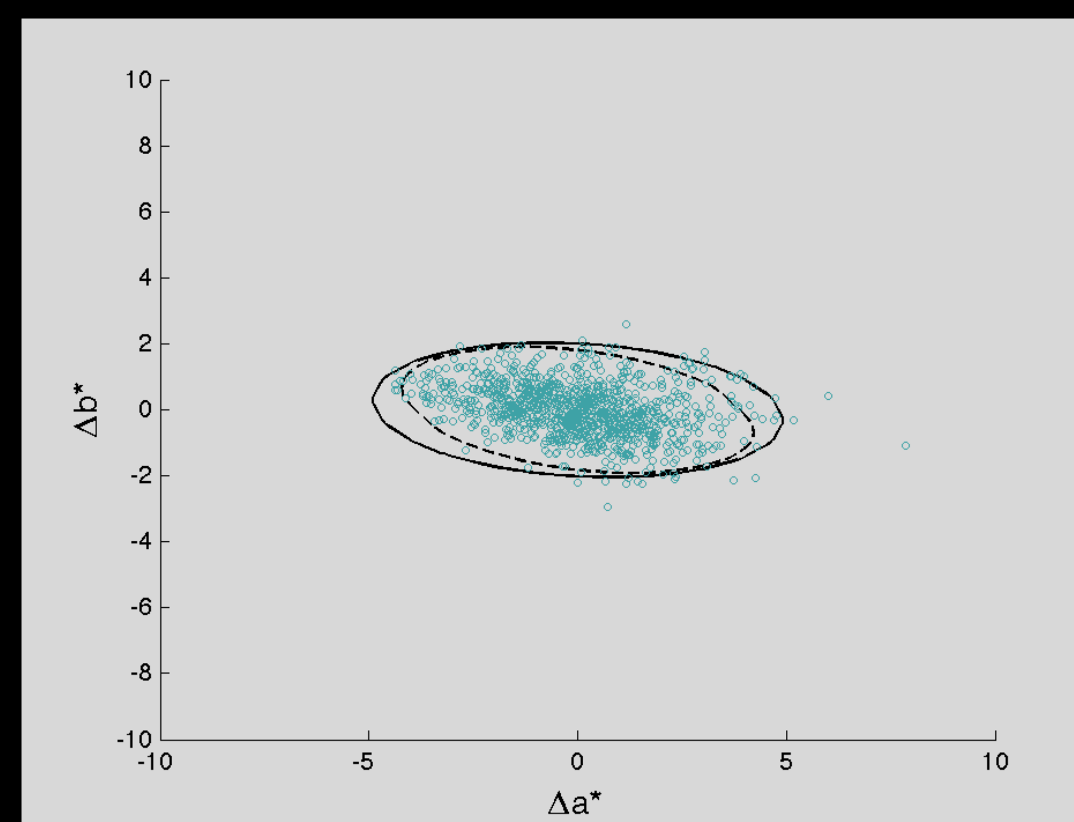
Mean and Variance (+Covariance CMFs)



Metameric Matches: 95 % Measured and Predicted



ASANO ISOCHROMATIC PLATES

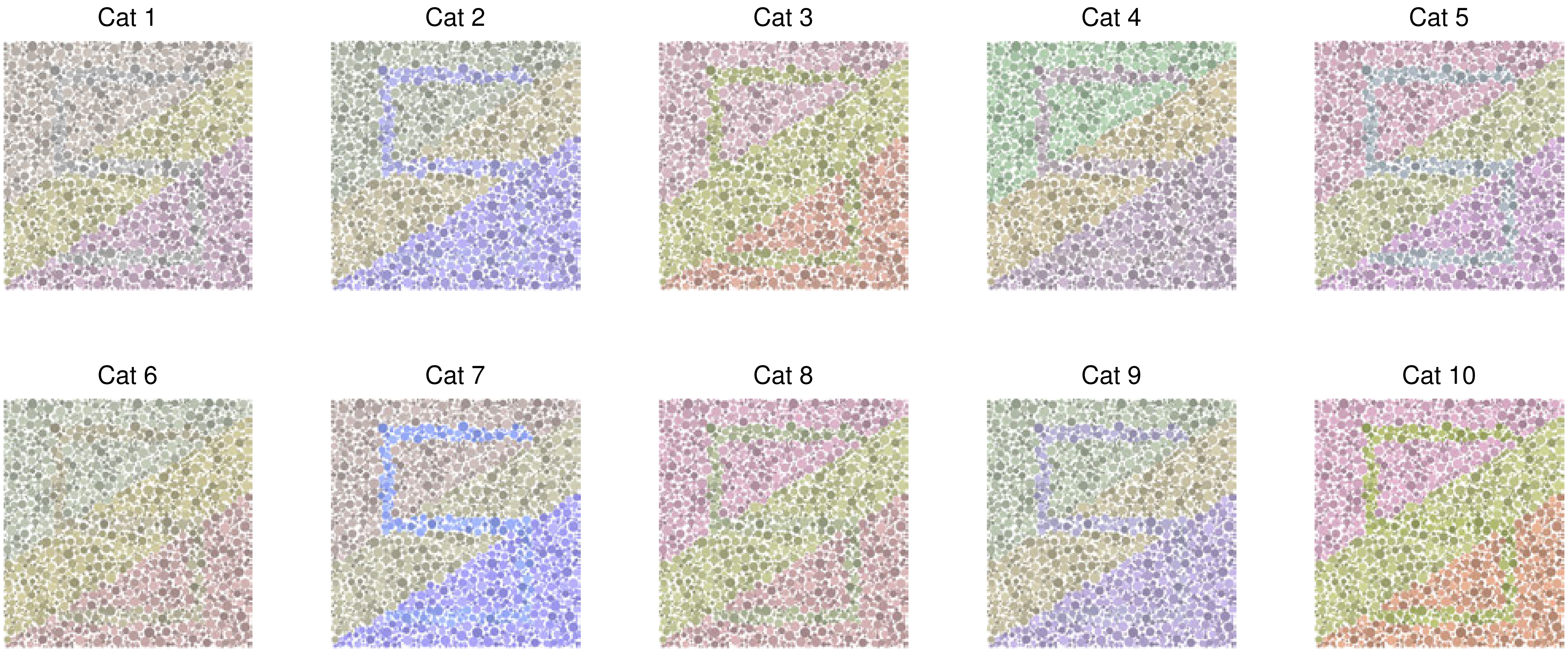


Fig. 4.9 – Visualization results of a spectral pseudoisochromatic image targeted at categorical observer 5, perceived by each of the ten categorical observers.

CONCLUSIONS / FUTURE

- Individualized Colorimetry (Custom CMFs)
- Complete Colorimetry (Nimeroff *et al.*)

- Here … Now … (Almost)
- Applications: *e.g.* cinema with laser projectors

THANK YOU...